# Quantum Circuits IV

# The Quantum Computing Stack

| | |
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| 5 | Applications |
| 3 | Algorithms |
| 2 | Circuits |
| 4 | Hardware |
| 1 | Mechanics |

Now Getting our hands dirty w/ circuits

Last Time

$2^+$ Qubits

# States and Gates

## Covered

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$X|0\rangle = |1\rangle$ $\qquad$ $H|0\rangle = |+\rangle$

$X|1\rangle = |0\rangle$ $\qquad$ $H|1\rangle = |-\rangle$

$$CNOT\,|b\rangle \otimes |a\rangle = |b \oplus a\rangle \otimes |a\rangle$$

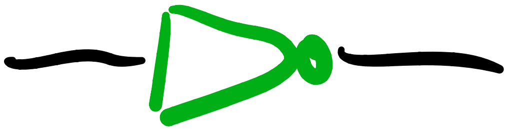
$1 \quad 1 \quad = 0$

$1 \quad 0 \quad = 1 \qquad 0\,1 = 1$

$0 \quad 0 \quad = 0$

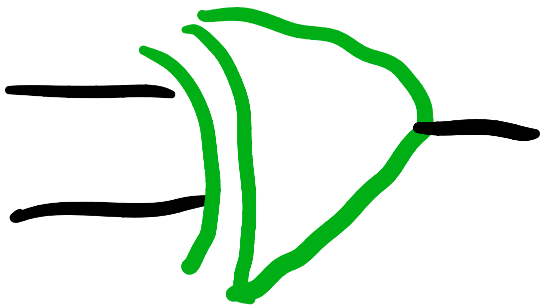
# Classical Gates

$A$



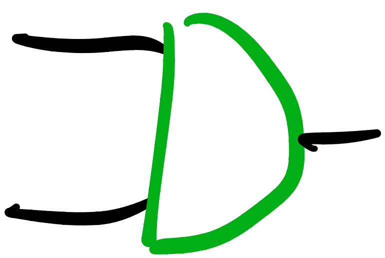
$\neg A$ — NOT

$A$, $B$



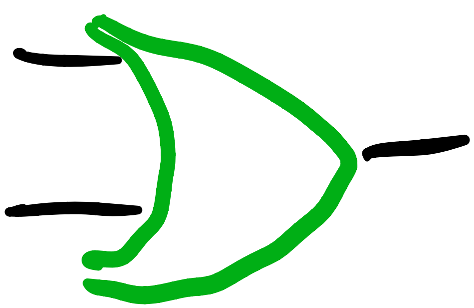
$A \oplus B$ — XOR

$A$, $B$



$A \wedge B$ — AND

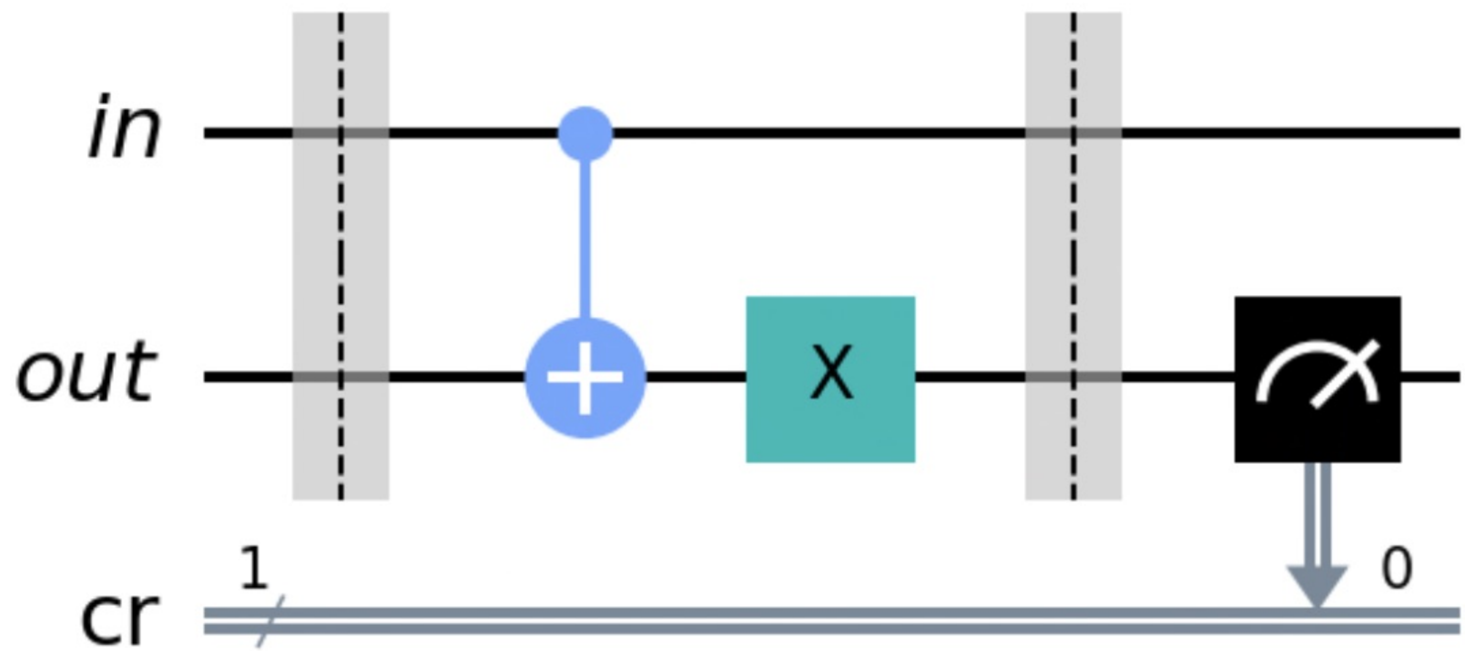
$A$, $B$



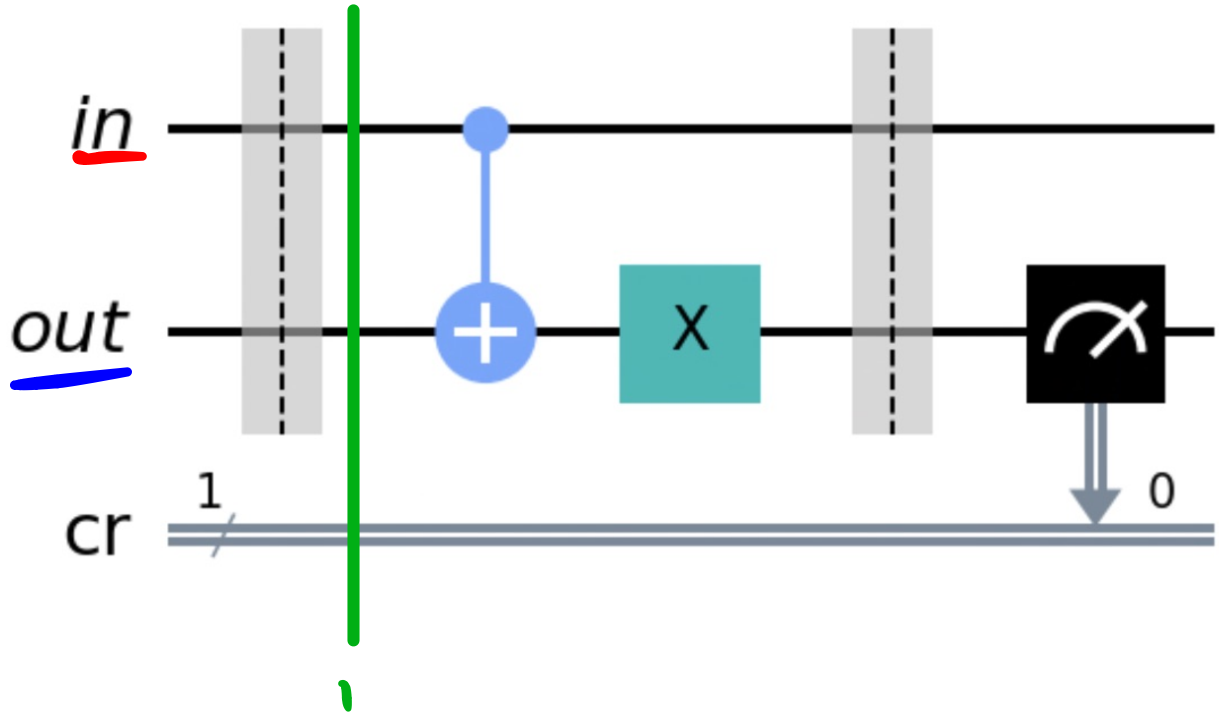
$A \vee B$ — OR

NOT

| in | out |
| --- | --- |
| 0 | 1 |
| 1 | 0 |



Reading 2 qubit Circuit

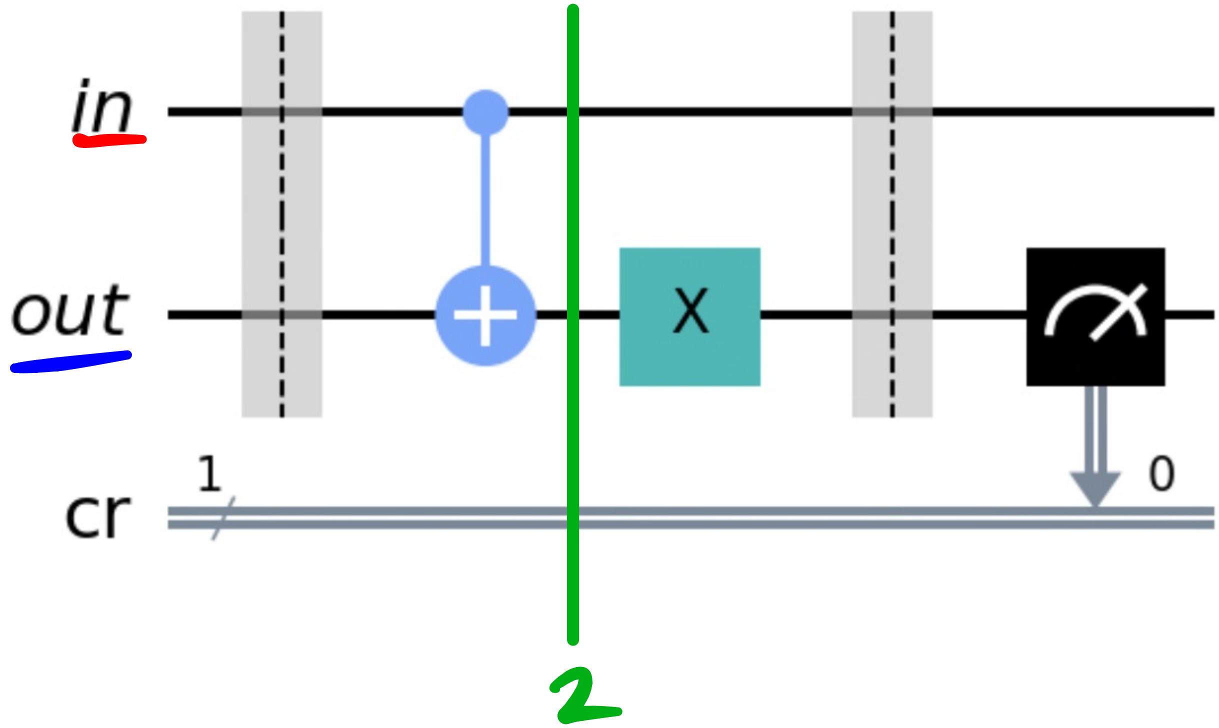



1:

Convenient Notation

$$|0\rangle \otimes |0\rangle = |00\rangle$$

Reading 2 qubit Circuit

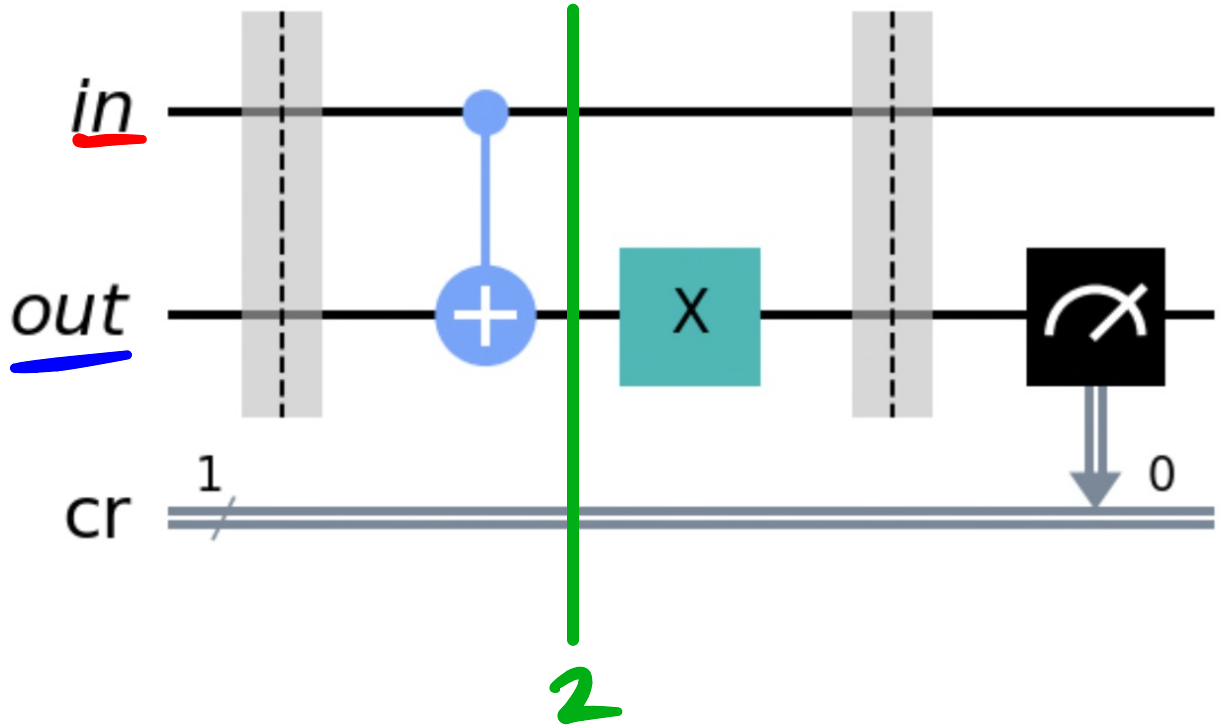



2:

$$\mathrm{CNOT}\left(|0\rangle \otimes |0\rangle\right) = |0 \oplus 0\rangle \otimes |0\rangle$$

$$= |0\rangle \otimes |0\rangle$$

Reading 2 qubit Circuit

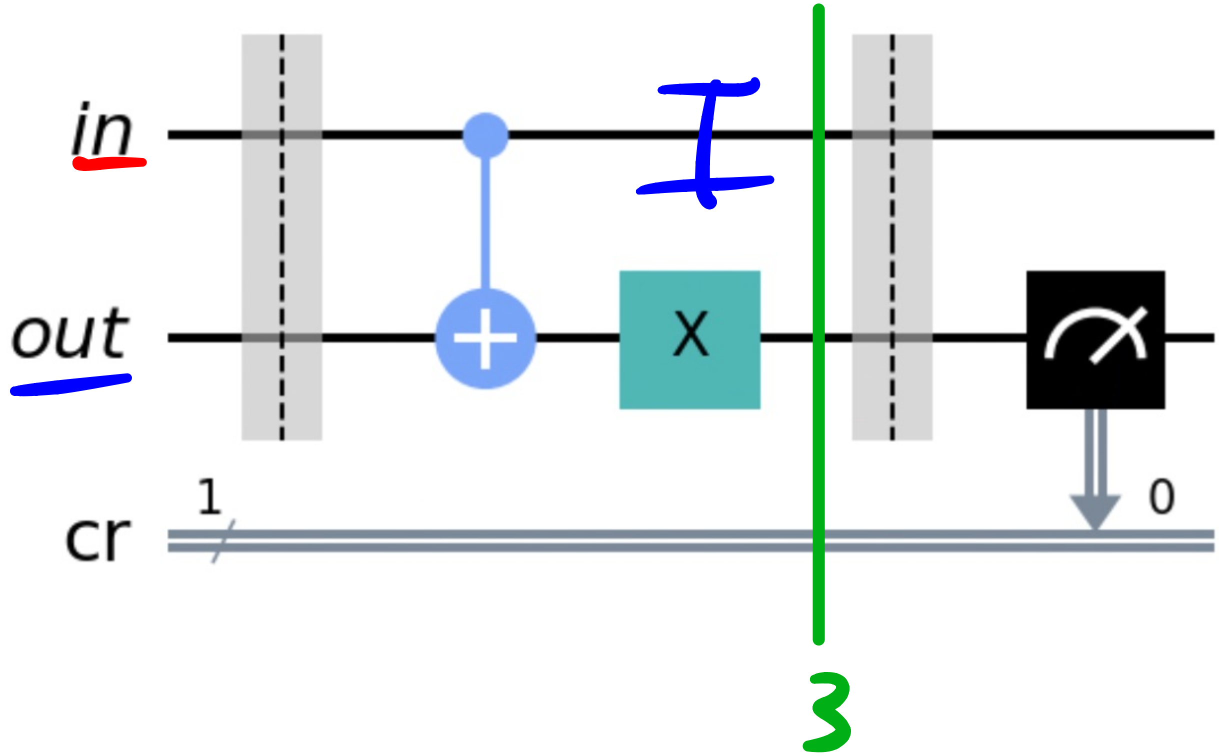



2:

$$CNOT\left(|0\rangle \otimes |0\rangle\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \otimes |0\rangle$$

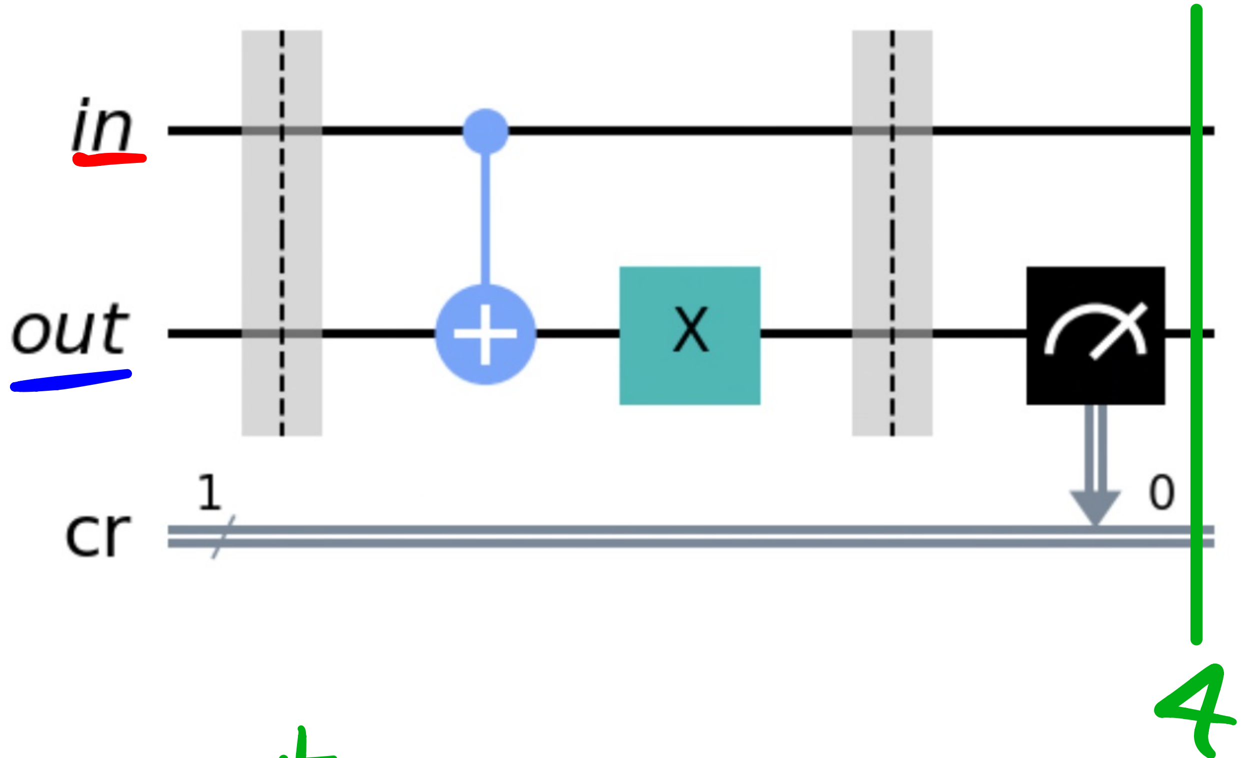
Reading 2 qubit Circuit




3:
$$(X \otimes I)(|0\rangle \otimes |0\rangle)$$

$$= X|0\rangle \otimes |0\rangle = |1\rangle \otimes |0\rangle = |10\rangle$$

Reading 2 qubit Circuit




Measurement

$|1\rangle \otimes |0\rangle$

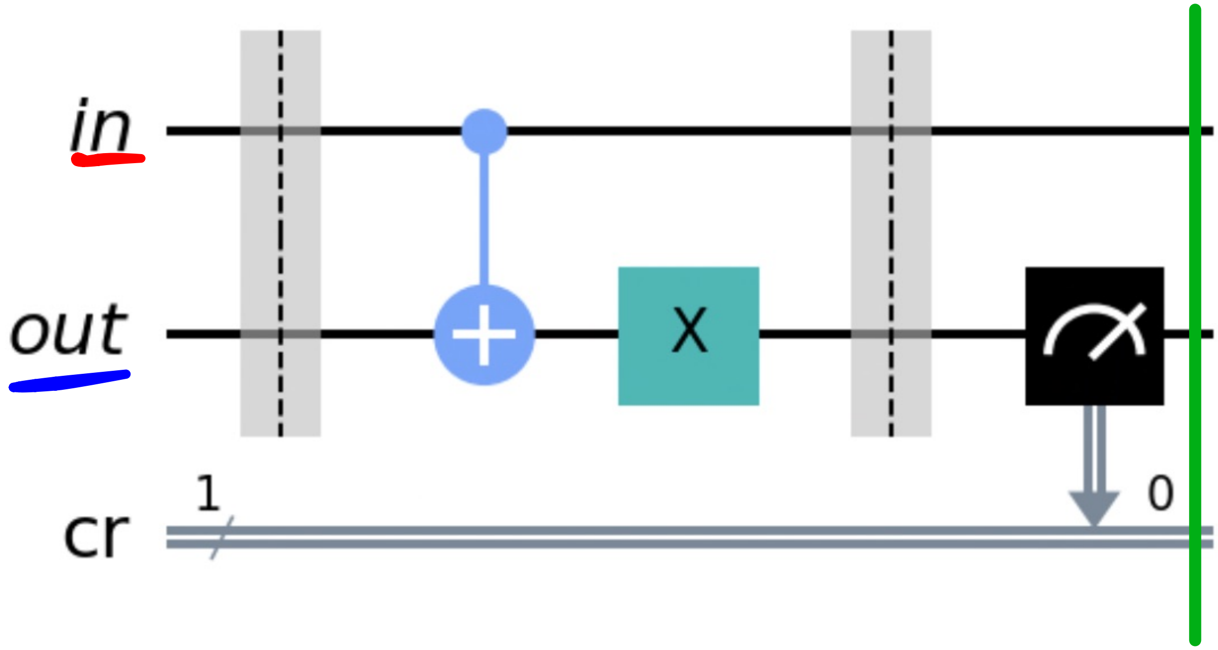
We only care to measure out

0:

$\langle 0|1\rangle = 0$

1:

$\langle 1|1\rangle = 1$




$$\text{circuit} = (X \otimes I)\,\text{CNOT}$$

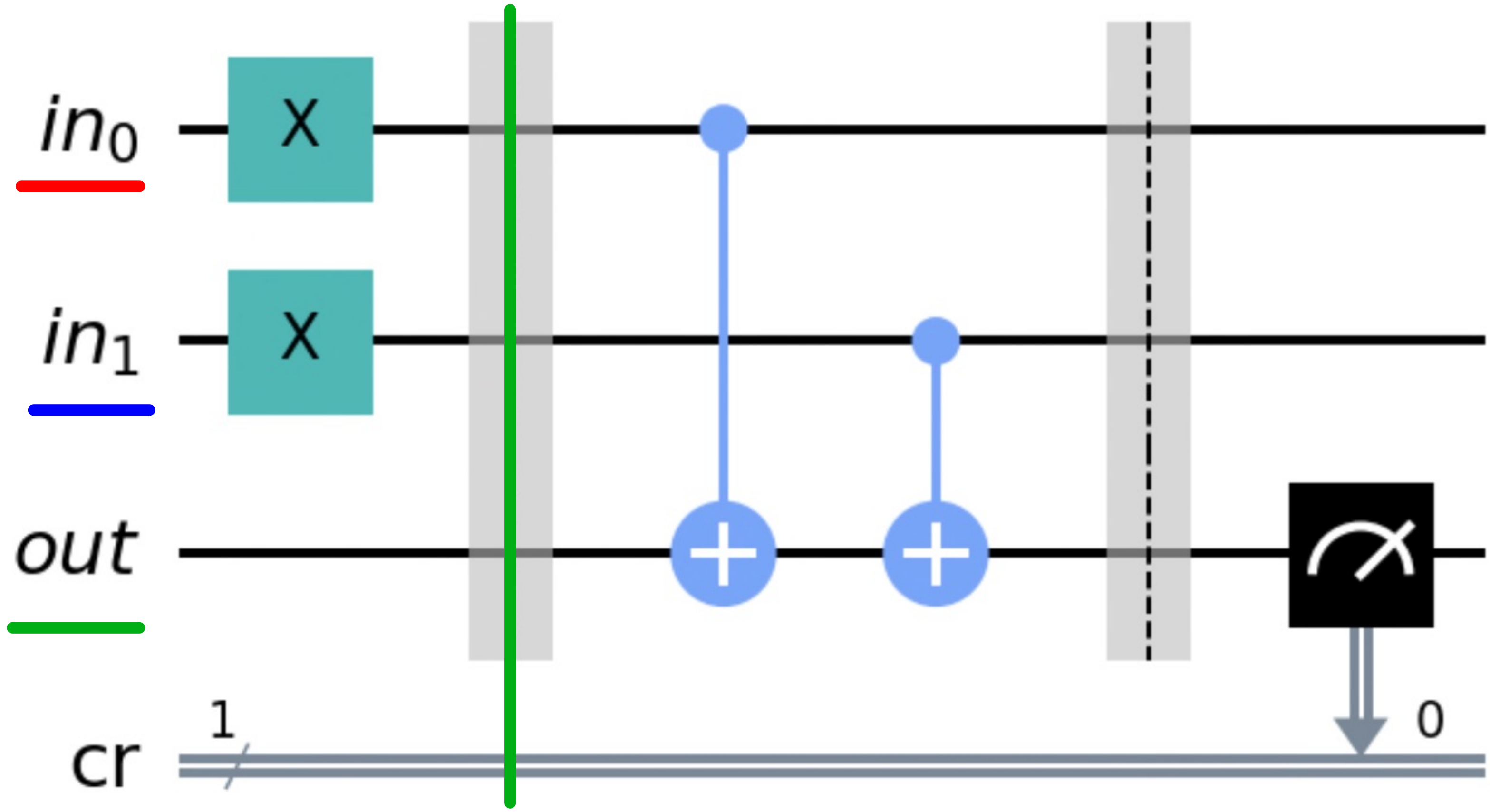
$$= \begin{bmatrix} \hat{0} & I \\ I & \hat{0} \end{bmatrix}$$

XOR

| $in_1$ | $in_0$ | out |
| --- | --- | --- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

CNOT $q_0 |1\rangle \rightarrow q_1 |1\rangle$

$$in_0 \oplus in_1 = out$$

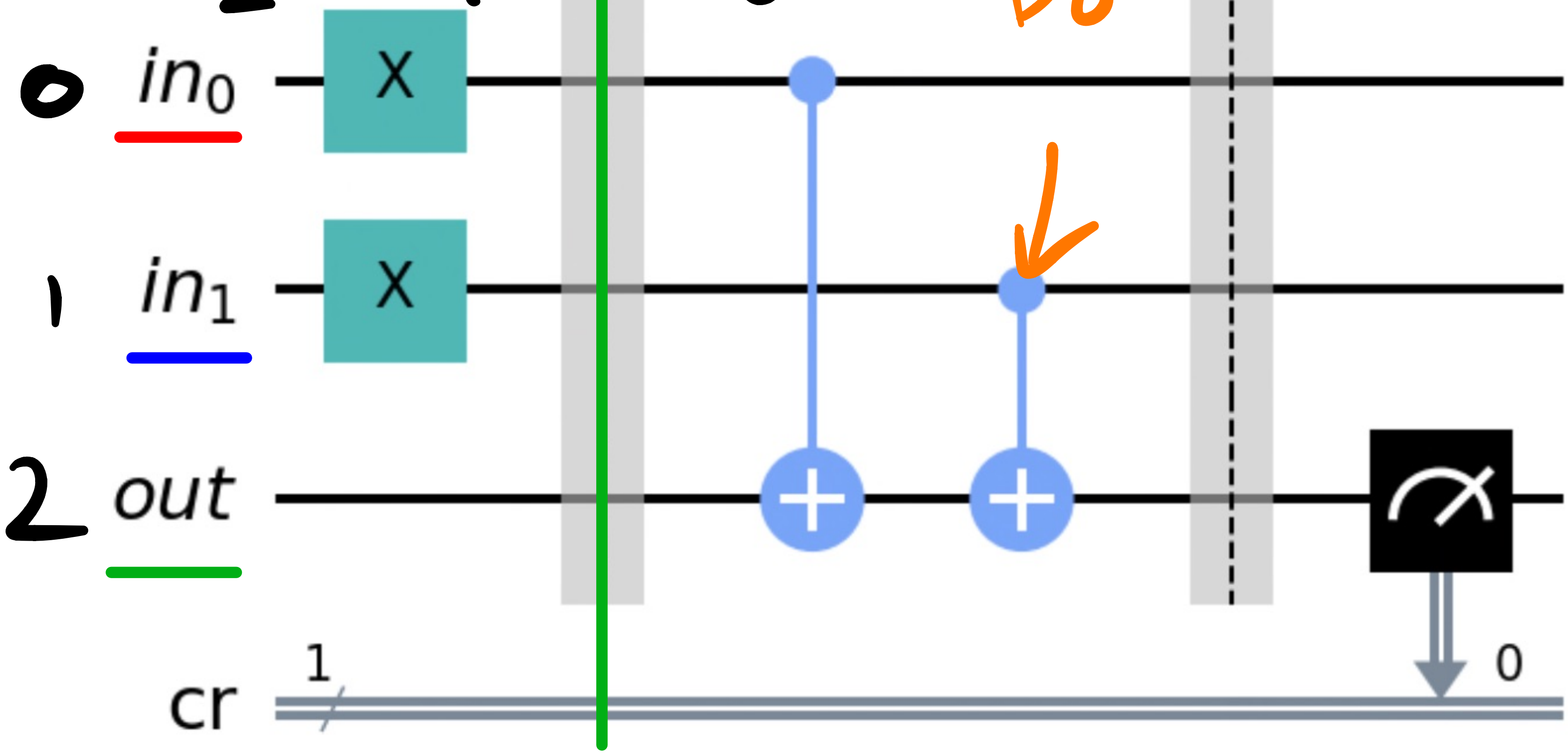



$|0\rangle \otimes |1\rangle \otimes |1\rangle$

$$|0\rangle \otimes |1\rangle \otimes |1\rangle$$

2 1 0

Do this!

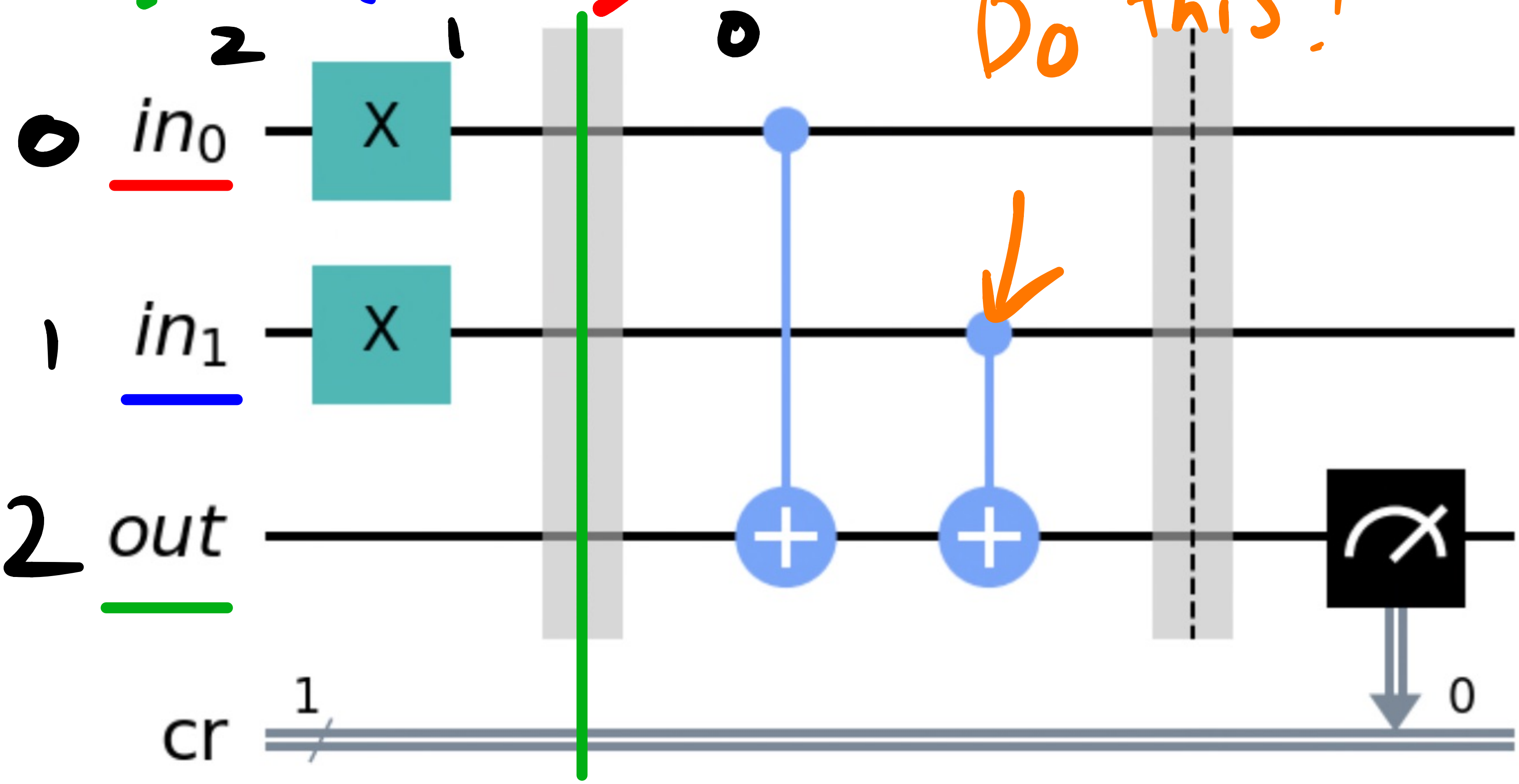



$$\text{CNOT}\left(|0\rangle \otimes |1\rangle\right) = |0 \oplus 1\rangle \otimes |1\rangle$$

$$= |11\rangle$$

Now: $|1\rangle \otimes |1\rangle \otimes |1\rangle$

$|1\rangle \otimes |1\rangle \otimes |1\rangle$




CNOT $\left(|1\rangle \otimes |1\rangle\right) = |1 \oplus 1\rangle \otimes |1\rangle$

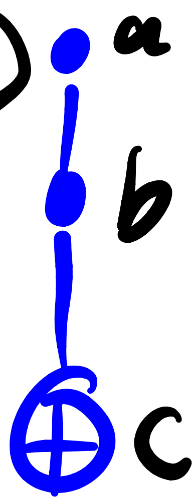
$= |01\rangle$

Now:

| $in_1$ | $in_0$ | out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

AND

$|c\rangle \otimes |b\rangle \otimes |a\rangle$




Toffoli

$$\mathrm{Tof}\left(|c\ b\ a\rangle\right) = |c \oplus (a \wedge b)\rangle \otimes |b\ a\rangle$$

OR

| $in_1$ | $in_0$ | out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

# Half-Adder

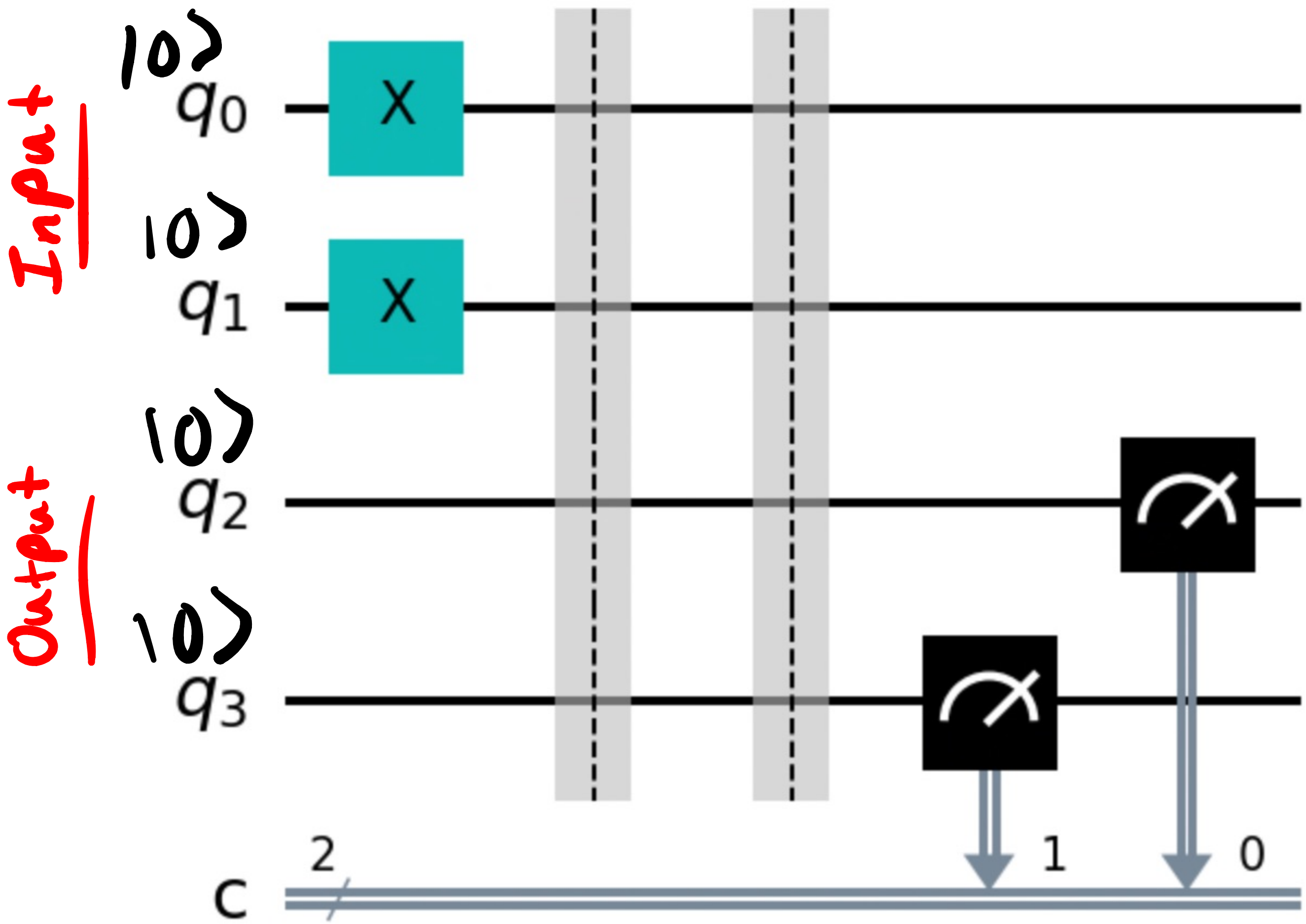
Binary

| $in_1$ | $in_0$ | $out_1$ | $out_0$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$in_1 + in_0 = \underbrace{out_1}_{\text{And}}\ \underbrace{out_0}_{\text{XOR}}$$

# Half-Adder Skeleton

States Gates Measurement




Why are th 'x' Gates in the states portion?

Keyword: Totoro

# Full - Adder

| $in_2$ + | $in_1$ + | $in_0$ = | $out_1$ | $out_0$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |